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Applications of Electromagnetic Waves to Problems

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14. ABSTRACT

This grant is primarily concerned with the development of new methods in nondestructive testing through the use of the theory of transmission eigenvalues. This investigation also includes an effort to extend the linear sampling method to the time domain in order to handle limited aperture data.

The main accomplishments during the period of this report were:

1. The derivation of new methods in nondestructive testing using the theory of transmission eigenvalues.
2. The introduction and investigation of a new class of inverse scattering problems that arise when sources and measurements are situated in the interior of a cavity.
3. The extension of the linear sampling method from the frequency domain to the time domain for penetrable inhomogeneous media.

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nondestructive testing, transmission eigenvalues, linear sampling method

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Applications of Electromagnetic Waves to Problems in Nondestructive Testing and Target Identification

Final Performance Report

July 15, 2011 - July 14, 2014

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Objectives

This grant is primarily concerned with the development of new methods in nondestructive testing through the use of the theory of transmission eigenvalues. This investigation also includes an effort to extend the linear sampling method to the time domain in order to handle limited aperture data.

Accomplishments/New Findings

The main accomplishments during the period of this report were:

1. The derivation of new methods in nondestructive testing using the theory of transmission eigenvalues.

2. The introduction and investigation of a new class of inverse scattering problems that arise when sources and measurements are situated in the interior of a cavity.

3. The extension of the linear sampling method from the frequency domain to the time domain for the case of a penetrable inhomogeneous medium.

Status of Effort

During the period of this proposal particular attention was paid to the use of transmission eigenvalues in non-destructive testing. Under the influence of a collaborative effort with the Materials and Manufacturing Directorate at Wright-Patterson Air Force Base in Dayton, Ohio, much of this effort was focused on numerical experiments using synthetic data to show that transmission eigenvalues can be accurately determined from measured scattering data and that the first transmission eigenvalue depends monotonically on the effective permittivity constant of the medium as well as on the volume of possible cavities [14]. Of particular interest has been the case of a dielectric object situated on a perfectly conducting surface [22], [26]. The latter leads to a new class of transmission eigenvalue problems involving mixed boundary conditions. For this class of problems it was shown that transmission eigenvalues exist, form a discrete set and can be determined from near-field data. The monotonicity property of the first transmission eigenvalue was established and the application of this property to obtain estimates of the permittivity of the dielectric was demonstrated. In a separate direction, this set of ideas was also investigated for the case of elastic waves [12].

The transmission eigenvalue problem has also been investigated for an inhomogeneous medium containing a finite number of small inhomogeneities of different refractive index [16]. In particular, an asymptotic formula was obtained for the perturbations of the real transmission eigenvalues caused by the presence of these small inhomogeneities. In a related direction, an asymptotic approximation was obtained to the solution of the interior transmission problem associated with scattering problem for a perfectly conducting body coated with a thin dielectric layer [2], [17]. Both of these asymptotic results have application to problems in nondestructive testing.

A third problem that we have investigated is a new class of inverse scattering problems that arise when sources and measurements are situated in the interior of a cavity [1], [10],

[11], [25]. Such situations arise in nondestructive testing when it is desired to monitor the material properties of an object through the use of internal measurements, for example the testing of the structural integrity of a reactor from measurements made inside the reactor. In the case of inhomogeneous media this leads to the investigation of an unusual non-selfadjoint eigenvalue problem called the exterior transmission eigenvalue problem.

We have also extended the frequency domain linear sampling method to the case of the time domain. In particular, we have shown that is is possible to determine the support of a penetrable inhomogeneous medium from measurements of causal scattered waves by using a time domain version of the linear sampling method [21].

Personnel Supported

1. Faculty
   D. Colton, P. Monk and F. Cakoni (Principal Investigators)

2. Short - Term Visitors
   Houssem Haddar, Ecole Polytechique, France
   Bojan Guzina, University of Minnesota, USA
   Virginia Selgas, University of Coruna, Spain
   Antonello Tamburino, Michigan State University, USA
   Rainer Kess, University of Goettingen, Germany
   John Sylvester, University of Washington, USA

Interactions/Transitions

Professors Colton, Monk and Cakoni have attended numerous conferences and seminars as invited speakers both in this country and in Europe and Asia.

Publications (with abstract)

2011

1. F. Zheng, F. Cakoni and J. Sun, An inverse electromagnetic scattering problem for a cavity, *Inverse Problems* **27** (2011), paper 125002.

   Abstract: We consider the inverse electromagnetic scattering problem of determining the shape of a perfectly conducting cavity from measurement of scattered electric field due to electric dipole sources on a surface inside the cavity. We prove a reciprocity relation for the scattered electric field and a uniqueness theorem for the inverse problem. Then the near field linear sampling method is employed to reconstruct the shape of the cavity. Preliminary numerical examples are provided to show the viability of the method.

2012

2. F. Cakoni, A. Cossonniere and H. Haddar, Transmission eigenvalues for inhomogeneous media containing obstacles, *Inverse Problems and Imaging* **6**, (2012), 373-398.

   Abstract: We consider the interior transmission problem corresponding to the inverse scattering by an inhomogeneous (possibly anisotropic) media in which an impenetrable obstacle with Dirichlet boundary conditions is embedded. Our main focus is to understand the associated eigenvalue problem, more specifically to prove that the transmission eigenvalues form a discrete set and show that they exist. The presence of Dirichlet obstacle brings new difficulties to already complicated situation dealing with a non-selfadjoint eigenvalue problem. In this paper, we employ a variety of variational techniques under various assumptions on the index of refraction as well as the size of the Dirichlet obstacle.

3. F. Cakoni, D. Colton and H. Haddar, The interior transmission eigenvalue problem for absorbing media, *Inverse Problems* **28** (2012), paper 045005.

   Abstract: In the recent years the transmission eigenvalue problem has been extensively studied for non-absorbing media. In this paper we initiate the study of this problem for absorbing media. In particular we show that, in the case of absorbing media, transmission eigenvalues form a discrete set, exist for sufficiently small absorption and for spherically stratified media exist without this assumption. For constant index of refraction we also obtain regions in the complex plane where the transmission eigenvalues cannot exist and obtain a priori estimate for real transmission eigenvalues.

4. F. Cakoni, M. Di Cristo and J. Sun, A multistep reciprocity gap functional method for the inverse problem in a multi-layered medium, *Complex Variables and Elliptic Equations* **57** (2012), 261-276.

Abstract: We introduce a multistep reciprocity gap functional method to reconstruct the support of a target embedded in a multi-layered medium from near field measurements at a fixed frequency. The approach is based on the use of the standard reciprocity gap functional method for homogeneous background to strip the layers and recover the data at each interface by mean of the so-called interior transmission problem and then, at the last step, to reconstruct the support of the scatterer. The advantage of this approach is that it avoids computing the Green's function for multi-layered medium. Numerical examples are given, showing the performance of our reconstruction algorithm.

5. F. Cakoni and D. Colton, Inverse problems and imaging: past, present and future, *UNESCO-EOLSS* online encyclopedia (2012).

   Abstract: The purpose of this article is to give a brief survey of the field of inverse problems. However this is by no means an easy task since the field has experienced tremendous growth in the past fifty years covering areas as diverse as computerized tomography, synthetic aperture radar, geophysical prospecting and nondestructive testing. Since the solution of any inverse problem is to "invert" the model to recover useful information about the physical phenomena from the observed image, inverse problems by definition must also deal with the subject of imaging.

6. E. Darrigrand and P. Monk, Combining ultra-weak variational formulation and multilevel fast multipole method, *Applied Numerical Mathematics*, **62** (2012), 709-719.

   Abstract: Because of its practical significance, many different methods have been developed for the solution of the time-harmonic Maxwell equations in an exterior domain at higher frequency. Often methods with complimentary strengths can be combined to obtain an even better method. In this paper we provide a numerical study of a method for coupling of the Ultra-Weak Variational Formulation (UWVF) of Maxwell?s equations, a volume based method using plane wave basis functions, and an overlapping integral representation of the unknown field to obtain an exact artificial boundary condition on an auxiliary surface that can be very close to the scatterer. Combining the new algorithm with a multilevel fast multipole method we obtain an efficient volume based solver with an exact auxiliary boundary condition, but without the need for singular integrals.

7. Y.J. Leung and D. Colton, Complex transmission eigenvalues for spherically stratified media *Inverse Problems*, **28** (2012), paper 075005.

   Abstract: We investigate the existence of complex transmission eigenvalues for spherically stratified media in $\mathbb{R}^2$ and $\mathbb{R}^3$. In $\mathbb{R}^2$ with the index of refraction being constant, we show that there exists an infinite number of complex eigenvalues. In $\mathbb{R}^3$ and constant index of refraction, we show that if the index of refraction is an integer there are no complex eigenvalues, whereas if the index of refraction is a rational number, complex eigenvalues can exist. Under appropriate assumptions we also show that complex eigenvalues can exist for a spherically stratified variable index of refraction.

8. P. Monk and V. Selgas, Sampling type methods for an inverse waveguide problem, *Inverse Problems and Imaging* **6**, (2012), 709-747.

Abstract: We consider the problem of locating a penetrable obstacle in an acoustic waveguide from measurements of pressure waves due to point sources inside the waveguide. More precisely, we assume that we are given the scattered field and its normal derivative for any source point and receiver placed on a pair of surfaces known as the source and the measurement surfaces, respectively. A novel feature of this work is that the obstacle is allowed to touch the boundary of the pipe. We first analyze the associated interior transmission problem. Then, we adapt and analyze the Reciprocity Gap Method (RGM) and the Linear Sampling Method (LSM) to deal with the inverse problem. We also study the relationship between these two methods and provide numerical results.

9. P. Monk and J. Sun, Finite element methods for Maxwell's transmission eigenvalues, *SIAM Journal on Scientific Computing* **34**, (2012), B247-B264.

Abstract: The transmission eigenvalue problem plays a critical role in the theory of qualitative methods for inhomogeneous media in inverse scattering theory. Efficient computational tools for transmission eigenvalues are needed to motivate improvements to theory, and, more importantly, are parts of inverse algorithms for estimating material properties. In this paper, we propose two finite element methods to compute a few lowest Maxwell's transmission eigenvalues which are of interest in applications. Since the discrete matrix eigenvalue problem is large, sparse, and, in particular, non-Hermitian due to the fact that the problem is neither elliptic nor self-adjoint, we devise an adaptive method which combines the Arnoldi iteration and estimation of transmission eigenvalues. Exact transmission eigenvalues for balls are derived and used as a benchmark. Numerical examples are provided to show the viability of the proposed methods and to test the accuracy of recently derived inequalities for transmission eigenvalues.

10. H. Qin and D. Colton, The inverse scattering problem for cavities, *Applied Numerical Mathematics*, **62** (2012), 699-708.

Abstract: We consider the inverse scattering problem of determining the shape of a perfectly conducting cavity from sources and measurements placed on a curve inside the cavity. A uniqueness theorem is proved and the shape is reconstructed by using a modification of the linear sampling method. Numerical examples are provided showing the viability of our method.

11. H. Qin and D. Colton, The inverse scattering problem for cavities with impedance boundary condition, *Advances in Computational Mathematics* , **36** (2012), 157-174.

Abstract: We consider the inverse scattering problem of determining the shape of a cavity with impedance boundary condition from sources and measurements placed on a curve inside the cavity. It is shown that both the shape ∂D of the cavity and

the surface impedance λ are uniquely determined by the measured data and numerical methods are given for determining both ∂D and λ where neither one is known a priori. Numerical examples are given showing the viability of our method.

2013

12. C. Bellis, F. Cakoni and B. Guzina, Nature of the transmission eigenvalue spectrum for elastic bodies, *IMA Journal of Applied Mathematics* **78** (2013), 895-923.

    Abstract: This study develops a spectral theory of the interior transmission problem (ITP) for heterogeneous and anisotropic elastic solids. The importance of this subject stems from its central role in a certain class of inverse scattering theories (the so-called qualitative methods) involving penetrable scatterers. Although simply stated as a coupled pair of elastodynamic wave equations, the ITP for elastic bodies is neither selfadjoint nor elliptic. To help deal with such impediments, earlier studies have established the well-posedness of an elastodynamic ITP under notably restrictive assumptions on the contrast in elastic parameters between the scatterer and the background solid. Due to the lack of problem self-adjointness, however, these studies were successful in substantiating only the discreteness of the relevant eigenvalue spectrum, but not its existence. The aim of this work is to provide a systematic treatment of the ITP for heterogeneous and anisotropic elastic bodies that transcends the limitations of earlier treatments. Considering a broad range of material-contrast configurations (both in terms of elastic tensors and mass densities), this paper investigates the questions of the solvability of the ITP, the discreteness of its eigenvalues and, for the first time, of the actual existence of such eigenvalue spectrum. Necessitated by the breadth of material configurations studied, the relevant claims are established through the development of a suite of variational formulations, each customized to meet the needs of a particular subclass of eigenvalue problems. As a secondary result, the lower and upper bounds on the first transmission eigenvalue are obtained in terms of the elasticity and mass density contrasts between the obstacle and the background. Given the fact that the transmission eigenvalues are computable from experimental observations of the scattered field, such estimates may have significant potential toward exposing the nature (e.g. compliance) of penetrable scatterers in elasticity.

13. C. Bellis, M. Bonnet and F. Cakoni, Acoustic inverse scattering using topological derivative of far-field measurements-based L^2 cost functionals, *Inverse Problems*, **29** (2013), paper 075012.

    Abstract: Originally formulated in the context of topology optimization, the concept of topological derivative has also proved effective as a qualitative inversion tool for a wave-based identification of finite-sized objects. This approach remains, however, largely based on a heuristic interpretation of the topological derivative, whereas most other qualitative approaches to inverse scattering are backed by a mathematical justification. As an effort toward bridging this gap, this study focuses on a topological

derivative approach applied to the L^2-norm of the misfit between far-field measurements. Either an inhomogeneous medium or a finite number of point-like scatterers are considered, using either the Born approximation or a full-scattering model. Topological derivative-based imaging functionals are analyzed using a suitable factorization of the far-field operator, for each of the considered cases, in order to characterize their behavior and assess their ability to reconstruct the unknown scatterer(s). Results include the justification of the usual sign heuristic underpinning the method for (i) the Born approximation and (ii) full-scattering models limited to moderately strong scatterers. Semi-analytical and numerical examples are presented. Within the chosen framework, the topological derivative approach is finally discussed and compared to other well-known qualitative methods.

14. F. Cakoni, D. Colton and P. Monk, *Transmission Eigenvalue Analysis for NDE and Characterization*, Preliminary Technical Report, University of Delaware, May, (2013).

    Abstract: This research project was concerned with the potential use of transmission eigenvalues for the purpose of the nondestructive testing of dielectrics using microwaves. The goal is to be able to detect, in a quantitative manner, both possible changes in the material structure of the dielectric as well as the presence of voids or cavities. Using synthetic data is has been shown that transmission eigenvalues can be accurately determined from measured scattering data and that the first transmission eigenvalue depends monotonically on the effective permittivity constant of the medium as well as on the volume of possible cavities. The next step in this effort is to use real experimental data to see if these results will be useful in problems of nondestructive testing of interest to the Air Force.

15. F. Cakoni and H. Haddar, Transmission eigenvalues in inverse scattering theory, *Inside Out 2*, MSRI Publications (2013), 529-580.

    Abstract: In the past few years transmission eigenvalues have become an important area of research in inverse scattering theory with active research being undertaken in many parts of the world. Transmission eigenvalues appear in the study of scattering by inhomogeneous media and are closely related to non-scattering waves. Such eigenvalues provide information about material properties of the scattering media and can be determined from scattering data. Hence they can play an important role in a variety of inverse problems in target identification and nondestructive testing. The transmission eigenvalue problem is a non-selfadjoint and nonlinear eigenvalue problem that is not covered by the standard theory of eigenvalue problems for elliptic operators. This article provides a comprehensive review of the state-of-the art theoretical results on the transmission eigenvalue problem including a discussion on fundamental questions such as existence and discreteness of transmission eigenvalues as well as FaberKrahn type inequalities relating the first eigenvalue to material properties of inhomogeneous media. We begin our presentation by showing how the transmission eigenvalue problem appears in scattering theory and how transmission eigenvalues are determined from

scattering data. Then we discuss the simple case of spherically stratified media where it is possible to obtain partial results on inverse spectral problems. In the case of more general inhomogeneous media we discuss the transmission eigenvalue problem for various types of media employing different mathematical techniques. We conclude our presentation with a list of open problems that in our opinion merit investigation.

16. F. Cakoni and S. Moskow, Asymptotic expansions for transmission eigenvalues for media with small inhomogeneities, *Inverse Problems*, **29** (2013), paper 104014.

    Abstract: We consider the transmission eigenvalue problem for an inhomogeneous medium containing a finite number of diametrically small inhomogeneities of different refractive index. We prove a convergence result for the transmission eigenvalues and eigenvectors corresponding to media with small homogeneities as the diameter of small inhomogeneities goes to zero. In addition we derive rigorously a formula for the perturbations in the real transmission eigenvalues caused by the presence of these small inhomogeneities.

17. F. Cakoni, N. Chaulet and H. Haddar, On the asymptotics of a Robin eigenvalue problem, *C. R. Math. Acad. Sci. Paris*, **351** (2013), 517-521.

    Abstract: The considered Robin problem can formally be seen as a small perturbation of a Dirichlet problem. However, due to the sign of the impedance value, its associated eigenvalues converge point-wise to ? as the perturbation goes to zero. We prove in this case that Dirichlet eigenpairs are the only accumulation points of the Robin eigenpairs with normalized eigenvectors. We then provide a criterion to select accumulating sequences of eigenvalues and eigenvectors and exhibit their full asymptotic with respect to the small parameter.

18. F. Cakoni and R. Kress, Integral equation methods for the inverse obstacle problem with generalized impedance boundary condition *Inverse Problems*, **29** (2013), paper 015005.

    Abstract: Determining the shape of an inclusion within a conducting medium from voltage and current measurements on the accessible boundary of the medium can be modeled as an inverse boundary value problem for the Laplace equation. We present a solution method for such an inverse boundary value problem with a generalized impedance boundary condition on the inclusion via boundary integral equations. Both the determination of the unknown boundary and the determination of the unknown impedance functions are considered. In addition to describing the reconstruction algorithms and illustrating their feasibility by numerical examples, we also obtain a uniqueness result on determining the impedance coefficients.

19. D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory*, 3rd edition, Springer, New York, 2013.

    Abstract: The inverse scattering problem is central to many areas of science and technology such as radar and sonar, medical imaging, geophysical exploration and

nondestructive testing. This book is devoted to the mathematical and numerical analysis of the inverse scattering problem for acoustic and electromagnetic waves. In this third edition, new sections have been added on the linear sampling and factorization methods for solving the inverse scattering problem as well as expanded treatments of iteration methods and uniqueness theorems for the inverse obstacle problem. These additions have in turn required an expanded presentation of both transmission eigenvalues and boundary integral equations in Sobolev spaces. As in the previous editions, emphasis has been given to simplicity over generality thus providing the reader with an accessible introduction to the field of inverse scattering theory.Review of earlier editions: Colton and Kress have written a scholarly, state of the art account of their view of direct and inverse scattering. The book is a pleasure to read as a graduate text or to dip into at leisure. It suggests a number of open problems and will be a source of inspiration for many years to come.SIAM Review, September 1994 This book should be on the desk of any researcher, any student, any teacher interested in scattering theory.Mathematical Intelligencer, June 1994.

20. D. Colton and Y.J. Leung, Complex eigenvalues and the inverse spectral problem for transmission eigenvalues, *Inverse Problems*, **29** (2013), paper 104008.

    Abstract: We continue our investigation of complex eigenvalues of the interior transmission problem for spherically stratified media (Leung and Colton 2012 Inverse Problems 28 075005). In this paper we show that if complex transmission values exist for a spherically stratified medium with (normalized) support in $\{x : |x| \leq 1\}$ then they must lie in a strip containing the real axis. We also give a new and shorter proof of the result of Aktosun et al (2011 Inverse Problems 27 115004), showing that a knowledge of all the transmission eigenvalues (real and complex) uniquely determine the index of refraction provided $0 < \eta(r) < 1$ for $0 < r < 1$ and $\eta(0)$ is known.

21. Y. Guo, P. Monk and D. Colton, Toward a time domain approach to the linear sampling method, *Inverse Problems*, **29** (2013), paper 095016.

    Abstract: We consider an inverse scattering problem for time-dependent acoustic waves in an inhomogeneous medium. We wish to determine the support of the inhomogeneity from measurements of causal scattered waves by using the linear sampling method in the time domain. The performance of the algorithm is illustrated with several numerical examples, including the first examples of the use of the time domain linear sampling method in three dimensions.

22. P. Monk and V. Selgas, Transmission eigenvalues for dielectric objects on a perfect conductor, *Inverse Problems*, **29** (2013), paper 104007.

    Abstract: We present a new interior transmission problem arising when a dielectric structure sits on a perfect conducting plane. The problem has mixed boundary conditions. We discuss the forward problem, and then briefly formulate the standard near field linear sampling method (LSM) for the inverse problem of shape identification. Next we show that the new mixed transmission eigenvalue problem can be analyzed

by appropriate modifications to the standard theory of transmission eigenvalues. In particular this involves proving appropriate density and compactness results. We end with some numerical evidence which shows that the LSM can be used for this problem even if limited aperture of data is used. In addition we demonstrate that transmission eigenvalues can be determined from near field scattering data.

2014

23. F. Cakoni and D. Colton, *A Qualitative Approach to Inverse Scattering Theory*, Springer, New York, 2014.

    Abstract: Inverse scattering theory is an important area of applied mathematics due to its central role in such areas as medical imaging , nondestructive testing and geophysical exploration. Until recently all existing algorithms for solving inverse scattering problems were based on using either a weak scattering assumption or on the use of nonlinear optimization techniques. The limitations of these methods have led in recent years to an alternative approach to the inverse scattering problem which avoids the incorrect model assumptions inherent in the use of weak scattering approximations as well as the strong a priori information needed in order to implement nonlinear optimization techniques. These new methods come under the general title of qualitative methods in inverse scattering theory and seek to determine an approximation to the shape of the scattering object as well as estimates on its material properties without making any weak scattering assumption and using essentially no a priori information on the nature of the scattering object. This book is designed to be an introduction to this new approach in inverse scattering theory focusing on the use of sampling methods and transmission eigenvalues. In order to aid the reader coming from a discipline outside of mathematics we have included background material on functional analysis, Sobolev spaces, the theory of ill posed problems and certain topics in in the theory of entire functions of a complex variable. This book is an updated and expanded version of an earlier book by the authors published by Springer titled Qualitative Methods in Inverse Scattering Theory. Review of Qualitative Methods in Inverse Scattering Theory All in all, the authors do exceptionally well in combining such a wide variety of mathematical material and in presenting it in a well-organized and easy-to-follow fashion. This text certainly complements the growing body of work in inverse scattering and should well suit both new researchers to the field as well as those who could benefit from such a nice codified collection of profitable results combined in one bound volume. SIAM Review, 2006.

24. D. Colton and R. Kress, *Integral Equation Methods in Scattering Theory*, SIAM Classics, Philadelphia, 2014.

    Abstract: This classic book provides a rigorous treatment of the Riesz Fredholm theory of compact operators in dual systems, followed by a derivation of the jump

relations and mapping properties of scalar and vector potentials in spaces of continuous and Hölder continuous functions. These results are then used to study scattering problems for the Helmholtz and Maxwell equations. Readers will benefit from a full discussion of the mapping properties of scalar and vector potentials in spaces of continuous and Hölder continuous functions, an in-depth treatment of the use of boundary integral equations to solve scattering problems for acoustic and electromagnetic waves, and an introduction to inverse scattering theory with an emphasis on the ill-posedness and nonlinearity of the inverse scattering problem.

25. Y. Hu, F. Cakoni and J. Liu, The inverse problem for a partially coated cavity with interior measurements, *Applicable Analysis*, **93** (2014), 936-956.

    Abstract: We consider the interior inverse scattering problem of recovering the shape and the surface impedance of an impenetrable partially coated cavity from a knowledge of measured scatter waves due to point sources located on a closed curve inside the cavity. First, we prove uniqueness of the inverse problem, namely, we show that both the shape of the cavity and the impedance function on the coated part are uniquely determined from exact data. Then, based on the linear sampling method, we propose an inversion scheme for determining both the shape and the boundary impedance. Finally, we present some numerical examples showing the validity of our method.

26. F. Yang and P. Monk, The interior transmission problem for regions on a conducting surface, *Inverse Problems*, **30** (2014), paper 015007.

    Abstract: We consider the interior transmission problem corresponding to inverse scattering for a bounded isotropic dielectric medium lying on an infinite conducting surface. In particular, we investigate the 2D scalar case of this problem where, in the corresponding scattering problem, the dielectric medium is illuminated by time harmonic transverse-electric (TE) or transverse-magnetic (TM) polarized electromagnetic waves, respectively. In both cases we establish the Fredholm property for this problem and show that transmission eigenvalues exist and form a discrete set. We also derive Faber - Krahn type inequalities for the transmission eigenvalues. Numerical results for the TE and TM cases are given showing that real transmission eigenvalues can be found from near field data, although in some cases the accuracy requirements on the data is very stringent.